Ponderomotive forces and stimulated Compton scattering of free electrons in a laser field

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The relationship between ponderomotive forces and stimulated Compton scattering is investigated. It is shown that an electron interacting with a coherent classical light field experiences an action of both ponderomotive forces and forces arising from the stimulated Compton scattering. Both of these forces are shown to be expressed in terms of a single function interpreted as the ponderomotive potential. Ponderomotive forces are shown to exist only if both electron and light states have at least some degree of coherence; they disappear if either the electron wave function is a pure plane wave or the field state is a pure quantum-electrodynamical state with definite numbers of photons (a pure Fock state). Stimulated Compton scattering is shown to depend also on the degree of coherence of the light field and of the free-electron wave function. Specific calculations are carried out for the plane-focused stationary Gaussian beam and for the stationary evanescent wave. Critical fields are found at which ponderomotive forces and forces arising due to the stimulated Compton scattering become of the same order of magnitude. [S1063-651X(96)04912-4]

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I. INTRODUCTION

In principle, the concepts of ponderomotive forces (PFs) and stimulated Compton scattering (SCS) are pretty well known. These phenomena have many manifestations in various physical effects and some of them have been studied for quite a long time [1,2]. One of the most recently discovered and investigated manifestations of the PFs is their influence on the observed energy spectra of photoelectrons arising in the process of above-threshold ionization of atoms [3] (see also the lists of references in [4,5]). As for the phenomenon of SCS, it is well known, e.g., to play the role of one of the mechanisms of plasma heating in a laser field [6,7]. Another well-known manifestation of SCS is the Kapitza-Dirac effect [8], i.e., scattering of electrons by a standing light wave (see also Refs. [4, 9, 10]). Both these and many other phenomena in which PF and/or SCS are involved have been thoroughly investigated both theoretically and experimentally.

However, in our opinion, in spite of the extensive investigation that has been carried out in this manifold of phenomena, there is a fundamental problem that is not well understood enough. Briefly, this is the problem of the relationship between PFs and SCS. Indeed, it is well known that PFs, affecting the electron motion in vacuum in an inhomogeneous light field, are linear in the light intensity I. It is also known that SCS and the associated changes of electron energy and momentum are proportional to I^2. On the other hand, it is well known from quantum electrodynamics that, in the absence of any other particles except electrons and photons, the Compton scattering is the lowest-order process that is not forbidden by the energy- and momentum-conservation rules. In the case when stimulated emission is much more efficient than spontaneous emission, this means that, according to quantum electrodynamics, SCS is the lowest-order process that is not forbidden by the energy- and momentum-conservation rules and this is the process of second order in I. The quantum-electrodynamical diagram in Fig. 1 ($\sim I$) gives the probability amplitude of SCS. Its squared absolute value is of the order of I^2 and determines the lowest-order rate of transitions. Hence, from the point of view of quantum electrodynamics, even the existence of PFs, linear in I, looks like a paradox that requires resolution and explanation.

The problem of the relationship between PFs and SCS can be specified further by formulating questions that deserve clarification. Some of them are the following.

(i) Is there any deep difference between PFs and SCS, or should one consider them as different manifestations of some single, more general, phenomenon?

(ii) Can PFs and SCS coexist, or do they arise under significantly different physical conditions?

(iii) If PFs and SCS can exist together, then, under what conditions do they match each other, i.e., when do the forces of the first and second order in I become of the same order of magnitude?

(iv) What are the roles of quantum and classical features of both electrons and a light field that determine similarities and differences between PFs and SCS?

The problem outlined and the questions formulated are addressed in the present paper.

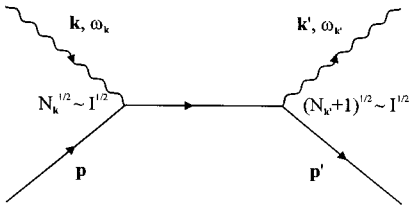

FIG. 1. Diagram of the process of stimulated Compton scattering; $\mathbf{p}$ and $\mathbf{p}'$ denote the electron momentum before and after scattering and $\mathbf{k}$ and $\mathbf{k}'$ denote wave vectors of absorbed and emitted photons.

II. DEFINITIONS

A. The light field

Let us begin from the definition of the method that will be used below to describe the light field. Let the field be characterized by its classical vector potential $\mathbf{A}(\mathbf{r},t)$, which can be expanded in the Fourier integral with expansion coefficients denoted as $\mathbf{A}_\mathbf{k}$:

$$\mathbf{A}(\mathbf{r},t)=\mathrm{Re}\left(\int d\mathbf{k}\,\mathbf{A}_\mathbf{k}\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]\right)$$
$$=\frac{1}{2}\left(\int d\mathbf{k}\{\mathbf{A}_\mathbf{k}\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]+\mathbf{A}_\mathbf{k}^*\exp[-i(\mathbf{k}\cdot\mathbf{r}-\omega t)]\}\right), \quad (1)$$

where $\omega_\mathbf{k}=c|\mathbf{k}|$ is the frequency corresponding to a wave vector $\mathbf{k}$. Below, only the squared vector potential will be needed in general. Let us assume that the frequency spectrum of the field $\mathbf{A}(\mathbf{r},t)$ is narrow enough $\Delta\omega\ll\omega_0$, where $\Delta\omega$ is the spectral width of light and ω_0 is its mean frequency. Then, the squared vector potential can be averaged over fast oscillations, i.e., the terms oscillating as $\exp(\pm 2i\omega t)$ can be dropped to give

$$\overline{\mathbf{A}^2(\mathbf{r},t)}=\tfrac{1}{2}\int d\mathbf{k}\,d\mathbf{k}'(\mathbf{A}_\mathbf{k}\cdot\mathbf{A}_{\mathbf{k}'}^*)\exp[i(\mathbf{k}-\mathbf{k}')\cdot r-i(\omega_\mathbf{k}-\omega_{\mathbf{k}'})t]. \quad (2)$$

In the case of a stationary field (long-pulse durations), the spectral width $\Delta\omega$ is so narrow that all the frequencies $\omega_\mathbf{k}$ and $\omega_{\mathbf{k}'}$ in Eq. (2) can be approximated by ω_0 to reduce Eq. (2) to the simplest form

$$\overline{\mathbf{A}^2(\mathbf{r})}=\tfrac{1}{2}\int d\mathbf{k}\,d\mathbf{k}'(\mathbf{A}_\mathbf{k}\mathbf{A}_{\mathbf{k}'}^*)\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}]. \quad (3)$$

To include in our consideration the case of a quantized field characterized by definite numbers of photons in the modes, we can use the same classical description as above, which has to be completed, however, by a transition to the so-called model of the δ-correlated field. This means that now all the classical Fourier components of the field $\mathbf{A}_\mathbf{k}$ are assumed to be some random functions obeying the averaging rule

$$\langle\mathbf{A}_\mathbf{k}\mathbf{A}_{\mathbf{k}'}^*\rangle=\frac{8\pi\hbar c^2 n_\mathbf{k}}{\omega_\mathbf{k}}\,\delta(\mathbf{k}-\mathbf{k}'), \quad (4)$$

where $n_\mathbf{k}$ is the number of photons per unit volume and per unit three-dimensional interval in the space of wave vectors, so that the total number density (the number of photons per unit volume) is given by $n_0=\int d\mathbf{k}\,n_\mathbf{k}$. If $N_\mathbf{k}$ denotes a total number of photons in the arbitrary mode $\mathbf{k}$, then, from the very-well-known relation $\Sigma_\mathbf{k}=[V/(2\pi)^3]\int d\mathbf{k}$ (where V is a large normalization volume $V\to\infty$), one can find easily that $n_\mathbf{k}=N_\mathbf{k}/(2\pi)^3$. For expressions proportional to the fourth power of the field, one has to complete the averaging rule of Eq. (4) by an assumption that an average product of four terms such as $\mathbf{A}_\mathbf{k}$ can be split into a sum of products of pairs of these functions

$$\langle(\mathbf{A}_\mathbf{k}\mathbf{A}_{\mathbf{k}'}^*)(\mathbf{A}_{\tilde{\mathbf{k}}}\mathbf{A}_{\tilde{\mathbf{k}}'}^*)\rangle=\langle\mathbf{A}_\mathbf{k}\mathbf{A}_{\mathbf{k}'}^*\rangle\langle\mathbf{A}_{\tilde{\mathbf{k}}}\mathbf{A}_{\tilde{\mathbf{k}}'}^*\rangle+\sum_{\alpha,\beta}\langle A_{\mathbf{k},\alpha}A_{\tilde{\mathbf{k}}',\beta}^*\rangle\times\langle A_{\mathbf{k}',\alpha}^*A_{\tilde{\mathbf{k}},\beta}\rangle$$
$$=[8\pi\hbar c^2]^2\left\{\frac{n_\mathbf{k}n_{\tilde{\mathbf{k}}}}{\omega_\mathbf{k}\omega_{\tilde{\mathbf{k}}}}\,\delta(\mathbf{k}-\mathbf{k}')\,\delta(\tilde{\mathbf{k}}-\tilde{\mathbf{k}}')+\frac{n_\mathbf{k}n_{\mathbf{k}'}}{\omega_\mathbf{k}\omega_{\mathbf{k}'}}\left|\mathbf{e}_\mathbf{k}\mathbf{e}_{\mathbf{k}'}^*\right|^2\delta(\mathbf{k}-\tilde{\mathbf{k}}')\,\delta(\mathbf{k}'-\tilde{\mathbf{k}})\right\}, \quad (5)$$

where $\mathbf{e}_\mathbf{k}$ is the polarization vector of the mode $\mathbf{k}$ and $\mathbf{A}_{\mathbf{k},\alpha}$ denotes the projection of the vector $\mathbf{A}_\mathbf{k}$ upon the axis α, $\alpha=1$, 2, and 3. The procedure described is convenient, though not necessary. In principle, a comparison with the quantum-electrodynamical case of pure Fock states can be made with the help of direct quantum-electrodynamical calculations. An example of the application of such an approach is discussed in Sec. V. However, the above-described procedure can be convenient for deriving quantum-electrodynamical expressions directly from the classical ones.

It should be noted that, in experiments, it can be rather difficult to realize pure Fock states of the field. On the other hand, the field of a multimode laser is an example of the field with almost uncorrelated phases of the modes. Such a field can be considered as an approximation to the field in a pure Fock state. Hence the results derived below for pure Fock states can be expected to be valid approximately for a field of multimode lasers. This expected similarity is a key point for an assumed experimental approach that can be used to check the below-derived results: instead of trying to construct a pure Fock state of a photon field, one can compare forces or electron declination angles in experiments with single-mode and multimode lasers. The last case has to give results closer to those derived below for pure Fock states rather than for a classical well-determined field with completely correlated phases of partial plane waves.

Below, the following two specific configurations of fields will be considered.

(i) The first is the plane-focused stationary Gaussian field [Fig. 2(a)] for which $\mathbf{A}_\mathbf{k}\|\mathbf{A}(\mathbf{r},t)\|0y$ and the functions $A_\mathbf{k}$ and $A(\mathbf{r},t)$ are given by

$$A_\mathbf{k}=k_0 d\sqrt{\frac{2}{\pi}}\frac{c\varepsilon_0}{\omega_0}\exp\left[-\frac{d^2}{2}[k_x^2+(k_z-k_0)^2]\right]\times\delta(k_y)\,\delta(k_x^2+k_z^2-k_0^2) \quad (6)$$

and

$$A(r,t)=\frac{c\varepsilon_0/\omega_0}{(1+z^2/L^2)^{1/4}}\exp\left[-\frac{x^2}{2d^2(1+z^2/L^2)}\right]\times\cos(k_0 z-\omega_0 t+\Phi), \quad (7)$$

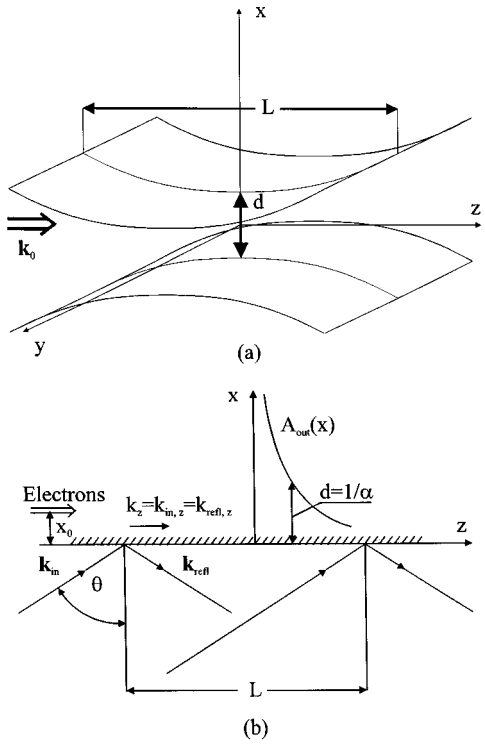

FIG. 2. Schemes of the field distributions: (a) plane-focused Gaussian laser beam and (b) evanescent plane wave.

where $k_0=c\omega_0$, ε_0 is the peak field-strength amplitude, d is the focal waist at $z=0$, $L=k_0d^2$ is the length of the focus, and Φ is a phase that is of no interest for our further consideration. The normalization coefficient in Eq. (6) is calculated in the paraxial approximation, which assumes that the distribution of wave vectors $\mathbf{k}$ around the z axis is narrow, i.e., that the angle χ between $\mathbf{k}$ and the z axis is small, $\chi\sim 1/dk_0\ll 1$.

In the case of a quantized field with definite numbers of photons in the modes (rigorously, in the above-described model of the δ-correlated field), one can also use an assumption about the Gaussian distribution of photons, under which the function $n_{\mathbf{k}}$ (a number of photons per unit volume and per unit three-dimensional interval in the space of wave vectors) is approximated by

$$n_{\mathbf{k}}=n_0\,\frac{2k_0d}{\sqrt{\pi}}\,\delta(k_y)\,\delta(k_x^2+k_z^2-k_0^2)\times\exp\{-d^2[k_x^2+(k_z-k_0)^2]\}. \tag{8}$$

For such a field, in the paraxial approximation ($\chi\ll 1$), the averaged squared vector potential is constant, i.e., independent of either time t or position vector $\mathbf{r}$,

$$\langle\overline{\mathbf{A}^2(\mathbf{r},t)}\rangle=\frac{8\pi\hbar c^2n_0}{\omega_0}\equiv\left(\frac{c\varepsilon_0}{\omega_0}\right)^2, \tag{9}$$

where, in this case, ε_0 is the effective field-strength amplitude corresponding to the total number density of photons n_0. So, in the case of a stationary beam of photons the average squared field is time independent and homogeneous, though the photon distribution over wave vectors [$n_{\mathbf{k}}$ of Eq. (8)] is inhomogeneous. In this case the parameter d in Eq. (8) cannot be interpreted as the beam waist or something like this. The parameter d can be interpreted only as the inverse width ($d\sim\Delta k^{-1}$) of the distribution over $\mathbf{k}$ or as the parameter determining the angular width of the photon beam $\Delta\chi\sim 1/dk_0$.

It should be emphasized again that, according to our assumption, the light field in a pure Fock state can be approximated to some extent by the field of a multimode laser. This assumption finds its confirmation in a well-known fact that the radiation field of a multimode laser can never be focused as well as the field of a single-mode laser: in the case of a multimode laser the minimal achievable focal waist $d_{\min}$ is much longer than the wavelength $\lambda_0=2\pi/k_0$. This conclusion agrees with Eq. (9), according to which, in the limiting case of a Gaussian beam of photons (a pure Fock state), the averaged squared field is absolutely homogeneous, i.e., independent of $\mathbf{r}$. Hence the ratio $d_{\min}/\lambda_0$ can be considered as a measure of the resemblance between the field of a multimode laser and that of a pure Fock state: the resemblance is greater the larger the parameter $d_{\min}/\lambda_0$.

(ii) The second specific field configuration considered below is that of a stationary evanescent wave that can arise under the conditions of the full reflection at the border between the vacuum and a medium with the refractive index $n>1$ [Fig. 2(b)]. In this case the averaged squared vector potential of the field is given by

$$\overline{\mathbf{A}^2(\mathbf{r},t)}=2\left(\frac{\varepsilon_0}{\omega_0}\right)^2\exp\left[-\left(\frac{z}{L}\right)^2\right]\times\begin{cases}\cos^2\left[xn\,\dfrac{\omega_0}{c}\cos(\theta)\right], & \text{inside}\\ \exp(-\alpha x), & \text{outside,}\end{cases} \tag{10}$$

where

$$\alpha=\frac{2\omega_0}{c}\sqrt{n\,\sin^2(\theta)-1} \tag{11}$$

and ''inside'' and ''outside'' indicate the regions inside and outside the denser medium and θ is the angle between the wave vector of the incident wave $\mathbf{k}_{\text{in}}$ and the x axis [see Fig. 2(b)]. The factor $\exp[-(z/L)^2]$ in Eq. (10) restricts the length of the region occupied by the incident, reflected, and evanescent waves in the z direction and L is the length of this region.

B. An electron

In our approach, an electron is assumed to be described quantum mechanically by its wave function $\Psi(\mathbf{r},t)$ obeying the Schrödinger equation

$$i\hbar\,\frac{\partial\Psi}{\partial t}=\frac{1}{2m}\left(-i\hbar\nabla-\frac{e}{c}\,\mathbf{A}(\mathbf{r},t)\right)^2\Psi(\mathbf{r},t). \tag{12}$$

In accordance with the idea about averaging over fast oscillations of the field (see its description in Sec. II A), such an averaging procedure can be applied directly to Eq. (12). As a result, in the time-averaged Schrödinger equation, the term linear in $\mathbf{A}(\mathbf{r},t)$ [$\propto(e/c)\mathbf{A}(\mathbf{r},t)i\hbar\nabla$] disappears, whereas in

the term proportional to the squared vector potential, the latter is replaced by $\overline{\mathbf{A}^2(\mathbf{r},t)}$ of Eq. (2) to give

$$i\hbar\,\frac{\partial\Psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\,\nabla^2+\frac{e^2}{2mc^2}\,\overline{\mathbf{A}^2(\mathbf{r},t)}\right]\Psi(\mathbf{r},t). \tag{13}$$

The second term in the square brackets on the right-hand side of Eq. (13) is often referred to as the ponderomotive potential

$$U_{\text{pond}}(\mathbf{r},t)=\frac{e^2}{2mc^2}\,\overline{\mathbf{A}^2(\mathbf{r},t)}. \tag{14}$$

In principle, the term linear in $\mathbf{A}(\mathbf{r},t)$ in Eq. (12), though being a fast oscillating function of time t, can give contributions to slowly varying components of the averaged Hamiltonian in the second order of iterations over fast oscillations. This results in some small corrections to the ponderomotive potential of Eq. (14). These corrections will be analyzed explicitly elsewhere, together with an analysis of relativistic effects. In this paper, only the nonrelativistic case is considered, i.e., the electron velocity v is assumed to be much smaller than the speed of light c, $v\ll c$, and the characteristic light frequency ω is also assumed to be not too large, $\hbar\omega\ll mc^2$.

In the classical approach, the PF is defined as the derivative of the ponderomotive potential

$$\mathbf{F}_{\text{pond,cl}}=-\boldsymbol{\nabla}U_{\text{pond}}(\mathbf{r},t). \tag{15}$$

Alternative names used for the PF in the classical mechanics of an electron in an inhomogeneous light field are the gradient force and the Gaponov-Miller force. The latter name is most often used in the plasma physics and originates from the papers [11] of the above-mentioned authors.

In the quantum-mechanical approach, the concept of PF originates equally naturally from the concept of the ponderomotive potential, although, in principle, as it will be shown below, the quantum-mechanical PF differs from $\mathbf{F}_{\text{pond,cl}}$ of Eq. (15). However, in quantum mechanics, the concept of ponderomotive potential is much more informative. In particular, the forces arising from SCS will be shown to be determined by the ponderomotive potential also.

In the general case, probably, the only method to solve the Schrödinger equation [Eq. (12)] analytically consists of using perturbation theory with respect to the ponderomotive potential. The arising results can be derived in the simplest way with the help of expansion of the wave function $\Psi(\mathbf{r},t)$ in plane waves

$$\Psi(\mathbf{r},t)=\int d\mathbf{p}\;C_{\mathbf{p}}(t)\,\psi_{\mathbf{p}}(\mathbf{r})\exp\left(-\frac{i}{\hbar}\,E_{\mathbf{p}}t\right), \tag{16}$$

where $E_{\mathbf{p}}=\mathbf{p}^2/2m$ and

$$\psi_{\mathbf{p}}(\mathbf{r})=\frac{1}{(2\pi\hbar)^{3/2}}\exp\left(\frac{i}{\hbar}\,\mathbf{p}\cdot\mathbf{r}\right). \tag{17}$$

In terms of perturbation theory, the wave function $\Psi(\mathbf{r},t)$ and its Fourier transform $C_{\mathbf{p}}(t)$ can be approximated by the first two terms of the corresponding expansions in powers of U_{pond},

$$\Psi(\mathbf{r},t)=\Psi^{(0)}(\mathbf{r},t)+\Psi^{(1)}(\mathbf{r},t)+\cdots \tag{18}$$

and

$$C_{\mathbf{p}}(t)=C_{\mathbf{p}}^{(0)}+C_{\mathbf{p}}^{(1)}(t)+\cdots\,, \tag{19}$$

where $C_{\mathbf{p}}^{(0)}=\text{const}$ and $C_{\mathbf{p}}^{(1)}(t)\sim\Psi^{(1)}(\mathbf{r},t)\sim U_{\text{pond}}$. The functions $C_{\mathbf{p}}^{(1)}(t)$ can be found in the usual way to have the form

$$C_{\mathbf{p}}^{(1)}(t)=-\frac{i}{\hbar}\int d\mathbf{p}'\,C_{\mathbf{p}'}^{(0)}\int_{-\infty}^{t}dt\langle\psi_{\mathbf{p}}(\mathbf{r})|U_{\text{pond}}(\mathbf{r},t')|\psi_{\mathbf{p}'}(\mathbf{r})\rangle \times\exp\left(-\frac{i}{\hbar}\,(E_{\mathbf{p}}-E_{\mathbf{p}'})t'\right). \tag{20}$$

Finally, as for the zeroth-order wave function $\Psi^{(0)}(\mathbf{r},t)$, the following two of its most characteristic forms will be used below: (i) the plane-wave unperturbed wave function

$$C_{\mathbf{p}}^{(0)}=\delta(\mathbf{p}-\mathbf{p}_0),\quad \Psi^{(0)}(\mathbf{r},t)=\frac{1}{(2\pi\hbar)^{3/2}}\exp\left(\frac{i}{\hbar}\,\mathbf{p}_0\cdot\mathbf{r}\right), \tag{21}$$

where $\mathbf{p}_0$ is the initial electron momentum, and (ii) the Gaussian wave packet

$$C_{\mathbf{p}}^{(0)}=\frac{(\Delta r_0)^{3/2}}{\pi^{3/4}}\exp\left[-\frac{1}{2\hbar^2}(\mathbf{p}-\mathbf{p}_0)^2(\Delta r_0)^2-\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}_0\right], \tag{22}$$

where Δr_0 is the size of the unperturbed wave packet and $\mathbf{r}_0$ is its position at $t=0$. Equations (16) and (22) yield

$$\Psi^{(0)}(\mathbf{r},t)=\frac{(\Delta r_0)^{3/2}}{\pi^{3/4}\left[(\Delta r_0)^2+i\,\dfrac{t\hbar}{m}\right]^{3/2}}\exp\left\{-\frac{(\mathbf{r}-\mathbf{r}_0)^2-2\,\dfrac{i}{\hbar}\,(\Delta r_0)^2\mathbf{p}_0\cdot(\mathbf{r}-\mathbf{r}_0)+\dfrac{it}{m\hbar}\,(\Delta r_0)^2p_0^2}{2\left[(\Delta r_0)^2+\dfrac{it}{m\hbar}\right]}\right\}. \tag{23}$$

The corresponding unperturbed electron density has the form of a spreading Gaussian distribution whose center of mass moves along the classical trajectory $\mathbf{r}_{\text{c.m.}}=\mathbf{r}_0+\mathbf{p}_0 t/m$:

$$|\Psi^{(0)}(\mathbf{r},t)|^2=\frac{1}{\pi^{3/2}[\Delta r(t)]^3}\left[-\frac{(\mathbf{r}-\mathbf{r}_0-\mathbf{p}_0 t/m)^2}{[\Delta r(t)]^2}\right], \tag{24}$$

where $\Delta r(t)$ is the time-dependent width of the spreading Gaussian wave packet

$$\Delta r(t)=\sqrt{(\Delta r_0)^2+\left(\frac{\hbar t}{m\Delta r_0}\right)^2}. \tag{25}$$

In contrast to the plane-wave unperturbed wave function [Eq. (21)], the Gaussian-packet wave function of Eqs. (23) and (24) is normalized by one.

In our specific calculations below (Sec. IV), the characteristic wave-packet spreading time

$$t_{\text{spr}}=\frac{(\Delta r_0)^2 m}{\hbar} \tag{26}$$

will be assumed to be much longer than other characteristic times of the problem under consideration, e.g., the time of flight of an electron through the focal region. However, in the arising formulas, the approximation $\Delta r(t)\approx\Delta r_0$ can be used very cautiously, typically, only in the final expressions after all the intermediate calculations have been completed.

III. FORCES: GENERAL EXPRESSIONS

In the framework of the quantum-mechanical approach, the force acting upon the electron can be defined as the rate of change of its average momentum

$$\mathbf{F}(t)=\frac{\overline{d\mathbf{p}(t)}}{dt}, \tag{27}$$

where

$$\overline{\mathbf{p}(t)}=\langle\Psi(\mathbf{r},t)|-i\hbar\boldsymbol{\nabla}|\Psi(\mathbf{r},t)\rangle. \tag{28}$$

Now, by using the Schrödinger equation [Eq. (12)] for the wave function $\Psi(\mathbf{r},t)$, without any approximations, one can transform Eqs. (27) and (28) to the form

$$\begin{aligned}\mathbf{F}(t)&=\int d\mathbf{r}|\Psi(\mathbf{r},t)|^2[-\boldsymbol{\nabla}U_{\text{pond}}(\mathbf{r},t)]\\&=\frac{i}{\hbar}\int d\mathbf{p}\,d\mathbf{p}'(\mathbf{p}'-\mathbf{p})\times C_{\mathbf{p}'}(t)C^*_{\mathbf{p}}(t)\\&\times\langle\psi_{\mathbf{p}}(\mathbf{r})|U_{\text{pond}}(\mathbf{r},t)|\psi_{\mathbf{p}'}(\mathbf{r})\rangle\\&\times\exp\left(-\frac{i}{\hbar}(E_{\mathbf{p}}-E_{\mathbf{p}'})t\right),\end{aligned} \tag{29}$$

where $\Psi(\mathbf{r},t)$ is an exact solution of Eq. (12) and $C_{\mathbf{p}}$ is its Fourier transform [Eq. (16)].

In the framework of perturbation theory, in analogy with Eqs. (18) and (19), the force $\mathbf{F}(t)$ can be presented in the form of a sum of the first- and second-order terms

$$\mathbf{F}(t)=\mathbf{F}^{(1)}(t)+\mathbf{F}^{(2)}(t)+\cdots. \tag{30}$$

The linear and squared dependence of $\mathbf{F}^{(1)}$ and $\mathbf{F}^{(2)}$ on the light intensity I can be considered as a sufficient motivation for interpretation of the first- and second-order forces $\mathbf{F}^{(1)}$ and $\mathbf{F}^{(2)}$ as the ponderomotive force and the force arising from SCS, respectively: $\mathbf{F}_{\text{pond}}=\mathbf{F}^{(1)}$ and $\mathbf{F}_{\text{SCS}}=\mathbf{F}^{(2)}$. It should be noted, however, that the ponderomotive force $\mathbf{F}_{\text{SCS}}$, defined in this way, coincides exactly with the classical ponderomotive force of Eq. (15) only under some specific conditions to be discussed below. Similarly, the force $\mathbf{F}_{\text{SCS}}=\mathbf{F}^{(2)}$ does not always correspond exactly to the well-known understanding of SCS used, e.g., in the theory of plasma heating [6,7]. So, the given definitions of $\mathbf{F}_{\text{pond}}$ and $\mathbf{F}_{\text{SCS}}$ can be considered as generalizations of the well-known and widely used concepts of PF and SCS. Specifically, $\mathbf{F}_{\text{pond}}$ and $\mathbf{F}_{\text{SCS}}$ are given by

$$\mathbf{F}_{\text{pond}}(t)\equiv\mathbf{F}^{(1)}(t)=\int d\mathbf{r}|\Psi^{(0)}(\mathbf{r},t)|^2[-\boldsymbol{\nabla}U_{\text{pond}}(\mathbf{r},t)] \tag{31}$$

and

$$\begin{aligned}\mathbf{F}_{\text{SCS}}(t)\equiv\mathbf{F}^{(2)}(t)=2\,\text{Re}\Bigg\{&\frac{i}{\hbar}\int_{-\infty}^{t}dt\int d\mathbf{p}\,\exp\left(\frac{i}{\hbar}E_{\mathbf{p}}(t-t')\right)\\&\times\langle\Psi^{(0)}(\mathbf{r}',t')|U_{\text{pond}}(\mathbf{r}',t')|\psi_{\mathbf{p}}(\mathbf{r}')\rangle\\&\times\langle\psi_{\mathbf{p}}(\mathbf{r})|-\boldsymbol{\nabla}U_{\text{pond}}(\mathbf{r},t)|\Psi^{(0)}(\mathbf{r},t)\rangle\Bigg\}.\end{aligned} \tag{32}$$

Expression (32) can be presented in some different forms that sometimes can be more convenient. One of these possibilities consists of performing integration over $\mathbf{p}$ in Eq. (32) to give

$$\begin{aligned}\mathbf{F}_{\text{SCS}}(t)=\frac{2}{\hbar^{5/2}}\left(\frac{m}{2\pi}\right)^{3/2}&\text{Re}\{(-\sqrt{i})\int_{-\infty}^{t}\frac{dt'}{(t-t')^{3/2}}\int d\mathbf{r}\,d\mathbf{r}'\\&\times[\Psi^{(0)}(\mathbf{r}',t')]^*U_{\text{pond}}(\mathbf{r}',t')\exp\left[-\frac{im}{2\hbar}\frac{(\mathbf{r}'-\mathbf{r})^2}{t-t'}\right]\\&\times[-\boldsymbol{\nabla}U_{\text{pond}}(\mathbf{r},t)]\Psi^{(0)}(\mathbf{r},t)\}.\end{aligned} \tag{33}$$

On the other hand, by substituting the expansion of the squared averaged vector potential [Eq. (2)] into the definition of the ponderomotive potential [Eq. (14)] and then into Eq. (32), one can transform the latter to the form

$$\begin{aligned}\mathbf{F}_{\text{SCS}}(t)=\tfrac{1}{2}\,\text{Re}\Bigg\{&\int_{-\infty}^{t}dt\int d\mathbf{p}\int d\mathbf{k}\,d\mathbf{k}'d\tilde{\mathbf{k}}\,d\tilde{\mathbf{k}}'(\tilde{\mathbf{k}}-\tilde{\mathbf{k}}')\\&\times\exp\Bigg[\frac{it'}{\hbar}(E_{\mathbf{p}}-E_{\mathbf{p}+\hbar(-\mathbf{k}+\mathbf{k}')})\\&+\frac{it}{\hbar}(E_{\mathbf{p}+\hbar(-\mathbf{k}+\mathbf{k}')}-E_{\mathbf{p}+\hbar(-\mathbf{k}+\mathbf{k}'-\tilde{\mathbf{k}}+\tilde{\mathbf{k}}')})\\&-it'(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'})-it(\omega_{\tilde{\mathbf{k}}}-\omega_{\tilde{\mathbf{k}}'})\Bigg](\mathbf{A}_{\mathbf{k}}\mathbf{A}^*_{\mathbf{k}'})(\mathbf{A}_{\tilde{k}}\mathbf{A}^*_{\tilde{\mathbf{k}}'})\\&\times(C^{(0)}_{\mathbf{p}})^*C^{(0)}_{\mathbf{p}+\hbar(-\mathbf{k}+\mathbf{k}'-\tilde{k}+\tilde{k}')}\Bigg\}.\end{aligned} \tag{34}$$

In the case of a stationary field, as mentioned above, all the frequencies in Eq. (34) ($\omega_{\mathbf{k}}$, $\omega_{\mathbf{k}'}$, $\omega_{\tilde{\mathbf{k}}}$, and $\omega_{\tilde{\mathbf{k}}'}$) can be substituted by the mean frequency ω_0.

Equation (34) is very convenient for the transition to the case of the field with δ-correlated modes (which is assumed to imitate the quantum-electrodynamical states of field with definite numbers of photons). By applying the averaging procedure of Eq. (5) to Eq. (34) we get

$$\langle \mathbf{F}_{\mathrm{SCS}} \rangle = 8\pi^3 c^4 r_0^2 \hbar^2 \int d\mathbf{k}\, d\mathbf{k}'(\mathbf{k}-\mathbf{k}')\, \frac{|\mathbf{e}_{\mathbf{k}} \cdot \mathbf{e}_{\mathbf{k}'}^*|^2}{\omega_{\mathbf{k}} \omega_{\mathbf{k}'}}\, n_{\mathbf{k}} n_{\mathbf{k}'} \times \int d\mathbf{p} |C_{\mathbf{p}}^{(0)}|^2 \delta[E_{\mathbf{p}+\hbar(-\mathbf{k}+\mathbf{k}')} - E_{\mathbf{p}} + \hbar(\omega_{\mathbf{k}} - \omega_{\mathbf{k}'})], \tag{35}$$

where $r_0 = e^2/mc^2$ is the classical electron radius.

Equation (35) has a clear physical interpretation as describing a superposition of processes in which photons are absorbed from a mode $\mathbf{k}'$ and emitted to a mode $\mathbf{k}$. The δ function under integration corresponds to the energy conservation rule for the total system ''electrons plus photons.'' The factor $\hbar(\mathbf{k}-\mathbf{k}')$ is the momentum acquired by an electron from photons via SCS. As a whole, Eq. (35) corresponds exactly to the diagram of Fig. 1. This fact can be considered as the confirmation that the model used of a stochastic field with δ-correlated modes imitates pretty well quantum-electrodynamical states of field with definite numbers of photons. The difference between Eqs. (32)–(34), on the one hand, and Eq. (35), on the other hand, is determined by the difference between a classical (coherent) field and incoherent field of quantum-electrodynamical states with definite numbers of photons. Equation (35) shows that in the quantum-electrodynamical limit coherence of the electron wave function, i.e., its wave-packet structure, does not play any role at all: the factor $|C_{\mathbf{p}}^{(0)}|^2$ does not depend on phases of $C_{\mathbf{p}}^{(0)}$ and is equivalent to the distribution function that would arise in the case of an incoherent momentum distribution of electrons.

It is interesting to check what happens with Eq. (35) if the wave-packet electron wave function is replaced by a plane wave of Eq. (21). In this case $C_{\mathbf{p}}^{(0)} = \delta(\mathbf{p}-\mathbf{p}_0)$ and the function $|C_{\mathbf{p}}^{(0)}|^2$ has a form of the squared singular δ function that has to be interpreted as

$$|C_{\mathbf{p}}^{(0)}|^2 = \delta(0)\,\delta(\mathbf{p}-\mathbf{p}_0) = \frac{1}{(2\pi\hbar)^{3/2}} \int d\mathbf{r}\ \delta(\mathbf{p}-\mathbf{p}_0) = \int d\mathbf{r} |\Psi^{(0)}(\mathbf{r},t)|^2 \delta(\mathbf{p}-\mathbf{p}_0). \tag{36}$$

The factor in front of the δ function on the right-hand side of Eq. (36) can be interpreted as the total number of electrons that is infinitely large because of the δ-function normalization of the wave functions of Eq. (21). In this case, the force $\mathbf{F}_{\mathrm{SCS}}$ of Eq. (35) no longer has the sense of the force acting on a single electron. To get such a one-electron force $\mathbf{f}_{\mathrm{SCS}}$, one has to divide $\mathbf{F}_{\mathrm{SCS}}$ by the number of electrons, i.e., just by the factor in front of the δ function on the right-hand side of Eq. (36). As a result, one gets

$$\langle \mathbf{f}_{\mathrm{SCS}} \rangle = 8\pi^3 c^4 r_0^2 \hbar^2 \int d\mathbf{k}\, d\mathbf{k}'(\mathbf{k}-\mathbf{k}')\, \frac{|\mathbf{e}_{\mathbf{k}} \cdot \mathbf{e}_{\mathbf{k}'}^*|^2}{\omega_{\mathbf{k}} \omega_{\mathbf{k}'}} \times n_{\mathbf{k}} n_{\mathbf{k}'} \delta(E_{\mathbf{p}+\hbar(-\mathbf{k}+\mathbf{k}')} - E_{\mathbf{p}} + \hbar(\omega_{\mathbf{k}} - \omega_{\mathbf{k}'})). \tag{37}$$

Formally, this result can be obtained from Eq. (35) by the substitution of $\delta(\mathbf{p}-\mathbf{p}_0)$ instead of $|C_{\mathbf{p}}^{(0)}|^2$.

The above-described procedure of transition to the δ-correlated field, or to the quantum-electrodynamical limit, can be applied also to Eq. (31) for the ponderomotive force $\mathbf{F}_{\mathrm{pond}}$. To do this, one has to express U_{pond} in terms of the squared vector potential $\mathbf{A}(\mathbf{r},t)$ [Eq. (14)], expand $\mathbf{A}(\mathbf{r},t)$ in plane waves [Eqs. (1) and (2)], and make a transition to the δ-correlating field by applying the averaging procedure of Eq. (4). As a result, one gets zero:

$$\langle \mathbf{F}_{\mathrm{pond}} \rangle = \mathbf{0}. \tag{38}$$

This means that in an incoherent quantized field with definite numbers of photons there are no ponderomotive forces at all. The only forces that can exist in this case are the forces originating from SCS. This result answers one of the questions formulated in the Introduction: whether or not the PF and SCS can coexist. There are two different answers to this question depending on the interpretation of the concept of SCS. The most conservative interpretation of SCS can be imagined to consist of the statement that, by definition, SCS is the process that occurs only if the unperturbed state of the field is a purely quantum-electrodynamical state with definite numbers of photons in the modes. Then, of course, the PF and SCS can never exist together. However, in our opinion, it is much more reasonable to interpret SCS as the effect that can occur in any fields, both incoherent and coherent (or partially coherent), including the purely coherent classical field. Of course, in this interpretation, SCS itself can depend on the degree of coherence of the field, and in Sec. IV such a dependence is demonstrated explicitly to be present. However, as for the PF and SCS, it is clear in the framework of such a wider interpretation that they can exist together only if the field is at least partially coherent, i.e., if its unperturbed state is given by a superposition of quantum-electrodynamical states with definite numbers of photons.

Another important conclusion concerns the case of a purely coherent classical field. As it is evident from Eq. (31), in this case, again, $\mathbf{F}_{\mathrm{pond}}=0$ if the unperturbed electron wave function $\Psi^{(0)}(\mathbf{r},t)$ is given by a plane wave [Eq. (21)]: for such a wave function $|\Psi^{(0)}(\mathbf{r},t)|^2 = \mathrm{const}$ and $\int d\mathbf{r}\,[-\nabla U_{\mathrm{pond}}(\mathbf{r},t)] = \mathbf{0}$. PF, $\mathbf{F}_{\mathrm{pond}}(t)$, are not equal to zero only if the unperturbed electron wave function has a form of a wave packet, i.e., is given by a coherent superposition of plane waves. Altogether, this result and the result formulated in the preceding paragraph can be summed up into a general conclusion that nonzero ponderomotive forces can exist only if both the field and the electron in their unperturbed states are coherent, at least partially. PFs turn zero if either the field or the electron are completely incoherent, i.e., if either the unperturbed state of the field is a purely quantum-electrodynamical state with definite numbers of photons or the electron wave function is a pure plane wave.

To conclude this section, before describing the results of specific calculations of $\mathbf{F}_{\mathrm{SCS}}$, it is worth discussing two cases when the forces arising from SCS turn zero. One of these cases is evident directly from the structure of Eq. (34). If the classical field is stationary ($\omega_{\mathbf{k}}=\omega_{\mathbf{k}'}=\omega_0$) and the electron wave packet is isotropic ($C_{\mathbf{p}}^{(0)}=C_{-\mathbf{p}}^{(0)}$), $\mathbf{F}_{\mathrm{SCS}}=\mathbf{0}$: the substitution $\mathbf{k}\leftrightarrow\mathbf{k}'$ and $\mathbf{p}\rightarrow-\mathbf{p}$ changes the sign of Eq. (34), though its structure remains unchanged. An isotropic wave packet corresponds to a particle that spreads but does not move, i.e., its center of mass remains at rest. For example, this is a feature of a photoelectron at the photoionization threshold, i.e., of an electron with zero kinetic energy.

The second of the two above-mentioned cases is the case of a classical field and of the unperturbed electron wave function $\Psi^{(0)}(\mathbf{r},t)$ having the form of a plane wave [Eq. (21)]. In contrast to the δ-correlated field, now the substitution of $\Psi^{(0)}(\mathbf{r},t)$ of Eq. (21) into Eq. (34) gives an absolutely regular expression with well-converging integrals of a product of two δ functions $\delta(\mathbf{p})$ and $\delta[\mathbf{p}+\hbar(-\mathbf{k}+\mathbf{k}'-\tilde{\mathbf{k}}+\tilde{\mathbf{k}}')]$ with non-coinciding arguments. Hence, in this case, although $\mathbf{F}_{\mathrm{SCS}}\neq 0$, $\mathbf{F}_{\mathrm{SCS}}$ is not infinitely large either. However, exactly as it was explained above, because of the δ-function normalization of $\Psi^{(0)}(\mathbf{r},t)$, the force acting on a single electron $\mathbf{f}_{\mathrm{SCS}}$ can be obtained from $\mathbf{F}_{\mathrm{SCS}}$ by dividing the latter by the infinitely large number of particles $\int d\mathbf{r}|\Psi^{(0)}(\mathbf{r},t)|^2=\infty$ and hence $\mathbf{f}_{\mathrm{SCS}}=\mathbf{0}$. It should be noted that in this specific case $\mathbf{F}_{\mathrm{pond}}=\mathbf{0}$ too and hence an electron does not experience an action of any forces at all, $\mathbf{f}_{\mathrm{SCS}}=\mathbf{f}_{\mathrm{pond}}=\mathbf{0}$, if it interacts with a classical field and its unperturbed wave function is given by a plane wave.

IV. RESULTS OF CALCULATIONS

A. The stationary classical Gaussian field

The calculations of the forces $\mathbf{F}_{\mathrm{pond}}$ and $\mathbf{F}_{\mathrm{SCS}}$ are performed with the help of Eqs. (6) and (7) for the classical Gaussian field, Eqs. (14) and (31) for the ponderomotive potential and force, Eqs. (22)–(25) for the Gaussian wave-packet electron wave function, and Eq. (34) for $\mathbf{F}_{\mathrm{SCS}}$. It is assumed that the field is stationary, i.e., that its pulse duration τ is longer than the main characteristic time of the problem that is given in our case by the time of flight of an electron through the focus t_{fl}, $\tau>t_{\mathrm{fl}}$. The electron momentum $\mathbf{p}_0$ will be assumed to be directed along the laser focal axis $(0z)$ and hence

$$t_{\mathrm{fl}}=mL/p_0, \tag{39}$$

where L is the length of the focus, and the condition that the field is stationary takes the form

$$\tau>mL/p_0. \tag{40}$$

As for the electron state, let us assume that its size Δr_0 is much smaller than the waist of the focus d,

$$\Delta r_0\ll d. \tag{41}$$

However, the size Δr_0 will be assumed to be large enough for the wave-packet spreading time t_{spr} [Eq. (26)] to be much longer than the time of flight t_{fl} [Eq. (39)],

$$\Delta r_0\gg(\hbar L/p_0)^{1/2}. \tag{42}$$

Under the formulated assumptions, the quantum-mechanical expression for the ponderomotive force [Eq. (31)] coincides with the classical one [Eq. (15)] because the electron density distribution in Eq. (31), $|\Psi^{(0)}(\mathbf{r},t)|^2$, can be approximated by the δ function $\delta(\mathbf{r}-\mathbf{r}_0-\mathbf{p}_0t/m)$. As for the force $\mathbf{F}_{\mathrm{SCS}}$ of Eq. (34), it is determined by a series of Gaussian integrals, the calculation of which is a rather cumbersome but straightforward procedure. Without dwelling upon any details of these calculations, let us describe here only the following two main results.

(i) By assuming that an electron moves exactly along the laser axis (with $x_0=0$ and $p_x=0$), we find that the force $\mathbf{F}_{\mathrm{SCS}}(t)$ is directed along the z axis and at $t=0$ in any point z_0 can be reduced to the form

$$F_{\mathrm{SCS},z}(z_0)=-\frac{1}{2}\left(\frac{e\varepsilon_0}{\omega}\right)^4\frac{Lz_0k_0^5}{m(k_0p_0)^2}\frac{k_0z_0+\sqrt{k_0^2z_0^2+k_0^2L^2}}{(k_0^2z_0^2+k_0^2L^2)^{3/2}}, \tag{43}$$

where, as previously, $k_0=c\omega_0$ is the mean wave vector of light. For $z_0\sim L$, Eq. (43) yields an estimate

$$F_{\mathrm{SCS},z}\sim-\left(\frac{e\varepsilon_0}{\omega}\right)^4\frac{k_0}{mp_0^2}\,\mathrm{sgn}(z_0). \tag{44}$$

Under the same conditions, the ponderomotive force of Eq. (15) is estimated as

$$F_{\mathrm{pond},z}\sim\left(\frac{e\varepsilon_0}{\omega}\right)^2\frac{1}{mL}\,\mathrm{sgn}(z_0). \tag{45}$$

The ratio of these forces is of the order of

$$\frac{F_{\mathrm{SCS},z}}{F_{\mathrm{pond},z}}\sim-\left(dk_0\,\frac{v_\varepsilon}{v_0}\right)^2, \tag{46}$$

where $v_\varepsilon=e\varepsilon_0^2/m\omega^2$ is the free-electron quiver motion peak velocity in the field, $v_0=p_0/m$ is the velocity of an incoming electron, and, as usual, dk_0 is a large factor $dk_0\gg1$.

The two conclusions that can be deduced from Eq. (46) sound interesting enough. First, the force arising from SCS is directed against the ponderomotive force. Second, these two forces become of the same order of magnitude, $F_{\mathrm{SCS},z}\sim F_{\mathrm{pond},z}$, when the field strength ε_0 achieves some critical value ε_c, where ε_c is determined by the condition

$$v_\varepsilon\approx\frac{v_0}{dk_0}. \tag{47}$$

Due to the large factor dk_0 in the denominator, the corresponding critical field ε_c cannot be too strong and, in addition, it depends on the field focusing geometry via the focal waist d, and this is rather unusual. Specifically, e.g., for $\omega=2\times10^{14}$ s^{-1} ($\lambda\equiv2\pi c/\omega\approx10^{-4}$ cm), $v_0=10^8$ cm/s, and $dk_0=10$, the critical field [the solution of Eq. (47)] is given by $\varepsilon_c\approx5\times10^8$ V/cm, which corresponds to the intensity $I_c\approx3\times10^{12}$ W/cm^2.

(ii)If we assume that an electron moves parallel to but at some distance $x_0\neq0$ from the focal axis $0z$, we can estimate in a similar way transverse forces $F_{\mathrm{SCS},x}$ and $F_{\mathrm{pond},x}$. The

corresponding general expression for $F_{\mathrm{SCS},x}$ is more complicated than Eq. (43) and, for this reason, is not reproduced here. But an estimate similar to Eq. (44) looks simple enough. At $t=0$ and $z_0=0$ (i.e., in the minimal-waist plane) and for $x_0 \sim d$, we get

$$F_{\mathrm{SCS},x} \sim -\left(\frac{e\varepsilon_0}{\omega}\right)^4 \frac{k_0^2 d}{m p_0^2}. \tag{48}$$

Under the same conditions, the ponderomotive force is estimated as

$$F_{\mathrm{pond},x} \sim \left(\frac{e\varepsilon_0}{\omega}\right)^2 \frac{1}{md}. \tag{49}$$

The ratio of these transverse forces $F_{\mathrm{SCS},x}/F_{\mathrm{pond},x}$ appears to be the same as the corresponding ratio of longitudinal forces [Eq. (46)]. Again, the force $F_{\mathrm{SCS},x}$ is directed against the corresponding ponderomotive force and they match each other under the same condition as that determined by Eq. (47). It is also interesting to notice that the transverse forces (both $F_{\mathrm{SCS},x}$ and $F_{\mathrm{pond},x}$) are larger than the corresponding longitudinal forces ($F_{\mathrm{SCS},z}$ and $F_{\mathrm{pond},z}$) by the same large factor $k_0 d \gg 1$.

B. Gaussian beam of photons

It can be interesting to compare the above-derived results arising in a classical field with the corresponding results occurring in the quantum-electrodynamical limit. Such a comparison can answer the question how the forces arising from SCS (and SCS itself) depend on a degree of coherence of both field and electrons. In accordance with the general discussion of Sec. IV A, in the case of a quantized field with definite numbers of photons in the initial state, the forces $\mathbf{F}_{\mathrm{SCS}}$ and $\mathbf{f}_{\mathrm{SCS}}$ [Eqs. (35) and (32)] do not depend on a coherence of the electron wave function even if such a coherence does occur. For this reason, it is most reasonable to estimate the one-electron force $\mathbf{f}_{\mathrm{SCS}}$ given by Eq. (37) in the case when the initial electron wave function is approximated by a plane wave. By substituting the Gaussian photon number density $n_{\mathbf{k}}$ [Eq. (8)] into Eq. (37) and calculating all the arising integrals, we finally get the result

$$f_{\mathrm{SCS},z}^{\mathrm{QED}} = -\left(\frac{e\varepsilon_0}{\omega}\right)^4 \frac{k_0}{2mp_0^2}. \tag{50}$$

By comparing this expression with Eq. (44), we find that $f_{\mathrm{SCS},z}^{\mathrm{QED}} \sim F_{\mathrm{SCS},z}^{\mathrm{cl}}$, i.e., longitudinal SCS forces arising in a classical and in a quantized fields are of the same order of magnitude.

An absolutely different situation occurs with transverse SCS forces. In a classical field, the force $F_{\mathrm{SCS},x}^{\mathrm{cl}}$ is given by Eq. (48). On the other hand, the calculation of a transverse SCS force in the case of a quantized field with definite numbers of photons gives $\mathbf{f}_{\mathrm{SCS},x}^{\mathrm{QED}} = \mathbf{0}$. This result can be easily understood qualitatively. Indeed, in accordance with Eq. (9), the averaged squared quantized field is homogeneous, i.e., independent of $\mathbf{r}$. Moreover, the photon-wave-vector distribution function $n_{\mathbf{k}}$ [Eq. (8)] is symmetric with respect to the substitution of x by the $-x$ direction (or k_x by $-k_x$). Hence, in this case, an electron moving along the z axis has no reason to prefer the x or the $-x$ direction, i.e., to get a momentum along a positive or negative direction of the x axis. So the above-derived result $f_{\mathrm{SCS},x}^{\mathrm{QED}} = 0$ can be considered as a direct consequence of these simple ideas about the symmetry of a quantized field. Contrarily, in a classical field, an electron moving in the z direction at some distance x_0 from the focal axis $x=0$ experiences an action of an inhomogeneous field. For such an electron there is no symmetry with respect to the substitution of x by $-x$, and for this reason $F_{\mathrm{SCS},x}^{\mathrm{cl}} \neq 0$.

In terms of coherence and incoherence concepts, the result derived ($f_{\mathrm{SCS},x}^{\mathrm{QED}} = 0$ and $f_{\mathrm{SCS},x}^{\mathrm{cl}} \neq 0$) can be interpreted as an indication that, under proper conditions, the coherence of light and electron states can emphasize the effect of SCS, whereas, vice versa, incoherence of a quantized field with definite numbers of photons can eliminate the effect (the force $f_{\mathrm{SCS},x}^{\mathrm{QED}}$ turns into zero).

Finally, it is worth remembering, in this context, the above-discussed idea that the field of a multimode laser can be considered as an approach to a pure Fock state, i.e., to the quantum-electrodynamical state with definite numbers of photons. As mentioned above, the field of a multimode laser is not homogeneous in space as it occurs in the case of Fock states. However, the minimal achievable size of a focus $d_{\min}$ is much larger in the case of multimode laser than in the case of a single-mode laser. For this reason, we expect that the transverse SCS force experienced by an electron interacting with the field of a multimode laser has to be somewhere between $F_{\mathrm{SCS},x}^{\mathrm{cl}}$ of Eq. (48) and $f_{\mathrm{SCS},x}^{\mathrm{QED}} = 0$. The difference between the maximal achievable transverse SCS forces in the cases of a single-mode and a multimode laser can be considered as a measure of incoherence of the radiation of a multimode laser, which partially eliminates the effect of SCS.

C. The stationary field of an evanescent light wave

Let us assume now that an inhomogeneous light field is formed by an evanescent light wave propagating along the surface of a medium with a sufficiently large refractive index. The corresponding averaged squared vector potential is given by Eqs. (10) and (11). Let us assume that all the conditions of Eqs. (39)–(42) are fulfilled, where now $d=1/\alpha$, with α given by Eq. (11). Moreover, to be sure that any direct interactions of incoming electrons with the medium can be ignored, we have to assume that electrons move along the surface of the medium (the plane $x=0$) parallel to the z axis at a distance x_0 from the surface [see Fig. 2(b)] with x_0 being much larger than the size Δr_0 of the electron wave packet

$$x_0 \gg \Delta r_0. \tag{51}$$

Under the formulated conditions, the calculations of $\mathbf{F}_{\mathrm{SCS}}$ were carried out with the help of Eqs. (10) and (33). With all the details of calculations dropped, the final result is given by

$$F_{\mathrm{SCS},x} = -\frac{1}{2}\left(\frac{e\varepsilon_0}{\omega}\right)^4 \frac{\alpha^3 L^2}{mp_0^2}. \tag{52}$$

Under the same conditions, the ponderomotive force $F_{\mathrm{pond},x}$, calculated with the help of Eq. (15) is equal to

$$F_{\mathrm{pond},x}=\left(\frac{e\varepsilon_0}{\omega}\right)^2\frac{\alpha}{m}. \tag{53}$$

As well as in the case of the Gaussian field, the force $F_{\mathrm{SCS},x}$ [Eq. (52)] is directed against the ponderomotive force $F_{\mathrm{pond},x}$ [Eq. (53)] and their ratio is given by

$$\frac{F_{\mathrm{SCS},x}}{F_{\mathrm{pond},x}}\approx-\frac{1}{2}\left(\alpha L\,\frac{v_\varepsilon}{v_0}\right)^2. \tag{54}$$

This estimate is practically identical to that of Eq. (46), where, for the Gaussian field, $k_0 d$ can be substituted by L/d.

V. COMPARISON WITH THE POTENTIAL SCATTERING THEORY

The above-derived results indicate, in particular, a rather significant difference between the two cases in which the electron wave function has the form of either a pure plane wave or a localized wave packet. This difference occurs even in the case of a stationary classical field, when the problem under discussion is equivalent to that of an electron scattering by a stationary ponderomotive potential $U_{\mathrm{pond}}(\mathbf{r})$. In accordance with the above-derived results, in this case, if the wave function of an incoming electron is a plane wave, any measurable scattering effects arise only in the second order in U_{pond}. Vice versa, if an incoming electron is localized and the size of its localization Δr_0 is smaller than the inhomogeneity size of $U_{\mathrm{pond}}(\mathbf{r})$, then the force $\mathbf{F}_{\mathrm{pond}}$ and the electron trajectory deviations arise already in the first order in U_{pond}. These results are general enough to be valid for any stationary potential $U(\mathbf{r})$ and they indicate a difference between the classical theory of potential scattering and the quantum-mechanical one (in its most-often found form with an incident electron described by a plane wave). Both in the classical and the quantum-mechanical theory of scattering, its main characteristics is the effective cross section $d\sigma$. In the classical theory $d\sigma$ is defined as [12]

$$d\sigma=\rho\,d\rho\,d\phi=\frac{\rho}{\sin(\theta)}\frac{d\rho}{d\theta}\,d\Omega, \tag{55}$$

where $d\Omega=\sin(\theta)d\theta\,d\phi$ is an element of a solid angle in the direction of motion of a scattered electron, whereas its initial velocity $\mathbf{v}_0$ is assumed to be directed along the z axis, ρ is the impact parameter, and θ and ϕ are the two spherical angles determining the direction of motion of a scattered electron. In the classical theory [12], the angle of scattering θ depends on the impact parameter ρ, $\theta=\theta(\rho)$. This dependence can be found explicitly in the approximation of small deviations of an electron from its original straight-line trajectory ($|\theta|\ll 1$). Under this assumption, for an electron in a spherically symmetric atomic potential $U(r)$, the Newton equation

$$m\ddot{\mathbf{r}}=-\boldsymbol{\nabla}U(r) \tag{56}$$

can be solved by iterations in $U(\mathbf{r})$. With the zeroth-order solution $\mathbf{r}^{(0)}=\boldsymbol{\rho}+\mathbf{v}_0 t$, in first-order Eq. (56) yields

$$\mathbf{F}^{(1)}(t)=m\ddot{\mathbf{r}}^{(1)}(t)=-\boldsymbol{\nabla}U(r)\big|_{r=\sqrt{\rho^2+v_0^2t^2}}. \tag{57}$$

Now, the scattering angle $\theta(\rho)$ is determined as the ratio of the first-order transverse momentum gained by an electron after scattering to its original longitudinal momentum mv_0,

$$\theta(\rho)=\frac{1}{mv_0}\left|\int_{-\infty}^{\infty}dt\;\mathbf{F}^{(1)}(t)\right| =\frac{1}{mv_0}\left|\frac{\partial}{\partial\rho}\int_{-\infty}^{\infty}dt\;U(\sqrt{\rho^2+v_0^2t^2})\right|. \tag{58}$$

For electrons scattered by an ensemble of atoms with the number density n_a, the average angle of scattering $\langle\theta\rangle$ is given by

$$\langle\theta\rangle=n_aL\int d\sigma\;\theta(\rho) =\frac{2\pi n_aL}{mv_0}\int_0^{\infty}\rho\,d\rho\left|\frac{\partial}{\partial\rho}\int_{-\infty}^{\infty}dt\;U(\sqrt{\rho^2+v_0^2t^2})\right|, \tag{59}$$

where L is the width of a layer of atoms in the z direction.

The most typical example of a model atomic potential is that of a screened Coulomb potential

$$U(r)=\frac{\gamma}{r}\exp(-\alpha r). \tag{60}$$

For atoms $\gamma=-Ze^2$, where $-Ze$ is the nucleon charge. For such a potential, Eq. (59) yields

$$\langle\theta\rangle=\frac{2\pi^2n_aL\gamma}{mv_0^2\alpha}. \tag{61}$$

Equations (58) and (68) indicate clearly a similarity to the concept of ponderomotive forces. The deviation angles $\theta(\rho)$ and $\langle\theta\rangle$ are of first order in the atomic potential $U(r)$.

These conclusions are different from those following from the standard quantum-mechanical theory of potential scattering. Indeed, the quantum-mechanical cross section of the electron-atom elastic scattering calculated in the first Born approximation is well known to have the form [13]

$$d\sigma=\frac{m^2}{4\pi^2\hbar^4}\left|\int d\mathbf{r}\;U(\mathbf{r})\exp\left[\frac{i}{\hbar}(\mathbf{p}-\mathbf{p}_0)\cdot\mathbf{r}\right]\right|^2 d\Omega, \tag{62}$$

where $\mathbf{p}_0=m\mathbf{v}_0$, $\mathbf{p}$ is the momentum of a scattered electron, $|\mathbf{p}|=|\mathbf{p}_0|$, as previously $d\Omega=\sin(\theta)d\theta\,d\phi$, $\mathbf{p}_0$ is directed along the z axis, and θ is the angle between $\mathbf{p}$ and $\mathbf{p}_0$. The average angle of scattering $\langle\theta\rangle$ is determined by the same equation as in the classical theory,

$$\langle\theta\rangle=n_aL\int\theta\,d\sigma. \tag{63}$$

For the specific case of a screened Coulomb potential [Eq. (60)] Eq. (63) yields

$$\langle\theta\rangle=\frac{2\pi^2n_aL\gamma^2}{mv_0^3\hbar\alpha}. \tag{64}$$

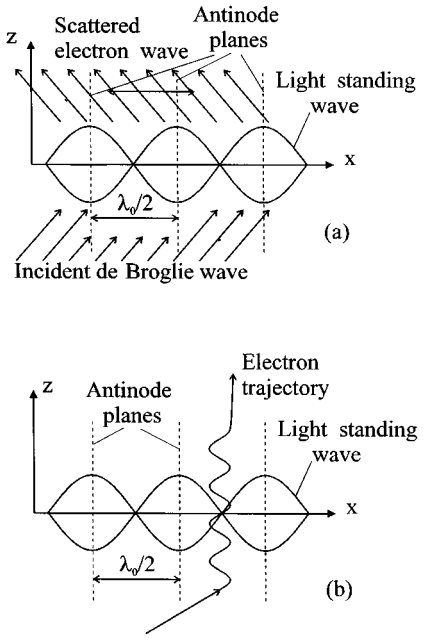

FIG. 3. Scattering of electrons by a standing light wave in the cases of electron wave functions given by (a) a plane de Broglie wave and (b) a localized wave packet.

The quantum-mechanical equations (62) and (64) are significantly different from the classical equations (59) and (61). In particular, in the case of a screened Coulomb potential, the ratio of the quantum-mechanical to classical average angles of scattering is given by

$$\frac{\langle\theta\rangle_{\mathrm{QM}}}{\langle\theta\rangle_{\mathrm{cl}}}=\frac{\gamma}{\hbar v_0}=\frac{Ze^2}{\hbar v_0}\ll 1, \tag{65}$$

where the last inequality follows from the quantum-mechanical applicability condition of the first Born approximation [13]. For any potential $U(\mathbf{r})$ the quantum-mechanical first Born average angle of scattering [Eq. (62)] is of second order in potential, whereas the corresponding classical average angle of scattering [Eq. (59)] is of first order in $U(r)$. In this sense, the classical result is equivalent to the above-discussed ponderomotive forces, whereas the quantum-mechanical formulas [Eqs. (62) and (64)] are reminiscent of the forces arising from SCS in a stationary classical inhomogeneous light field. Also, the conditions of realization of the classical and quantum-mechanical predictions in the theory of potential scattering are similar to the conditions of realization of the PF and SCS: the classical results of Eqs. (62) and (64) have to be valid if the electrons are well localized in space, close to their classical dotlike image. Vice versa, quantum-mechanical predictions have to be correct for wide wave packets or electron wave functions close to plane waves. It should be noted, however, that for the electron-atom scattering, it is rather difficult to realize well localized and long-living electron wave packets with sizes smaller than an atomic size. Vice versa, in the case of scattering from a focused light field, such a situation is quite realistic because the focal size is usually much larger than the atomic radius. Hence, practically, it is much easier to observe the effects of first order in the potential (PF) in the case of the electron-light interaction than in the case of electron-atom scattering. On the other hand, we assume that the classical effects of first order in $U(r)$ can become observable even in the usual experiments on scattering by atomic or molecular targets if electrons would be scattered by large-size macromolecules rather than by atoms of a normal atomic size.

A similar analysis can be given in such a specific and frequent case of the electron-light interaction as the scattering of electrons by a standing light wave (Fig. 3). If an electron is described quantum mechanically and its unperturbed wave function is a plane wave or a large-size wave packet ($\Delta r_0 \gg \lambda$, where λ is the light wavelength), the process of scattering can be interpreted in terms of SCS or in terms of diffraction of the electron de Broglie wave on the periodical structure of a standing light wave [Fig. 3(a)]. This interpretation agrees completely with that of the original paper by Kapitza and Dirac [8]. In this case, as it is usual for SCS; in the weak-field approximation, the forces acting upon an electron and the observable scattering probabilities are of the order of I^2, i.e., they are proportional to the second power of the light intensity I. If, however, the electron's wave packet is narrow ($\Delta r_0 \ll \lambda$), its trajectory [Fig. 3(b)] is similar to that of a classical particle channeling in a trough formed by neighboring antinode planes of the standing wave [14]. These two regimes are significantly different. In particular, in contrast to the above-mentioned case of a wide packet (or a plane-wave electron wave function), in the case of a narrow wave packet the parameters of the trough and the oscillation frequency of a channeling electron are determined by PFs, i.e., by forces of first order in the light intensity I [14]. This regime has to occur for any electron beams, wide or narrow, if only the single-electron wave functions are well localized, i.e., have the form of wave packets with sizes Δr smaller than λ.

VI. DISCUSSION

Let us summarize first the conditions under which the ponderomotive force and the force arising due to the stimulated Compton scattering exist or do not exist. These conditions are given by Tables I and II, in which ''classical'' and ''QED'' refer to states of the field, classical (coherent) and quantum electrodynamical, with definite numbers of photons (incoherent) and ''wave packet'' and ''plane wave'' indicate the initial electron wave function taken in the form of a wave packet or a plane wave, respectively. The signs + and − mean that the corresponding process exists (+) or does not exist (−).

Returning now to the ''paradox'' formulated in the Introduction, we can see that, in fact, this is not a paradox at all: in the case of a quantized field with definite numbers of photons in the initial state $\mathbf{F}_{\mathrm{pond}}=\mathbf{0}$, it is true that SCS is the lowest-order nonzero quantum-electrodynamical effect and the corresponding probabilities or rates are of the order of I^2. However, the above-mentioned paradox can be reformulated as the question, what is wrong in the argumentation, resulting in the conclusion that the lowest-order force is $\sim I^2$, in the case of a classical field and localized electron states. Let us repeat this argumentation: the lowest-order process is described by the diagram of Fig. 1. This diagram determines

TABLE I. Conditions of existence of the PF, F_{pond}.

Electron \ Field	Classical	QED
Wave packet	+	−
Plane wave	−	−

TABLE II. Conditions of existence of the forces arising from SCS, F_{SCS} or f_{SCS}.

Electron \ Field	Classical	QED
Wave packet	+	+
Plane wave	−	+

the probability amplitudes and is of the order of I. The squared absolute value of the diagram of Fig. 1 determines probabilities, rates of transitions, or rates of change of the electron energy or momentum, and they are all of the order of I^2. This sequence of statements is absolutely correct in the quantum-electrodynamical limit, i.e., if the initial electron state is a plane wave and the initial state of the field is a state with definite numbers of photons. So, what becomes wrong in the above-described chain of statements in the opposite, classical, limit? The answer is simple: in a classical field and for localized states of an electron *not only the squared absolute value of the diagram of Fig. 1*, but also the *diagram itself* can determine physically measurable characteristics of the process under consideration, e.g., the rate of change of the electron momentum. Indeed, a classical field can be considered as a quantum-electrodynamical field in a state that is given by a superposition of states with definite numbers of photons [15]. Similarly, an electron wave packet is a superposition of plane waves. If such superpositions are rich enough to include terms characterized by electron and field quantum numbers both before and after scattering ($\{\mathbf{p}, N_{\mathbf{k}}, \text{ and } N_{\mathbf{k}'}\}$ and $\{\mathbf{p}', N_{\mathbf{k}}-1, \text{ and } N_{\mathbf{k}'}+1\}$), the diagram of Fig. 1 itself, averaged over such an unperturbed state, gives a nonzero result. To make this conclusion clearer, it is reasonable to rewrite Eq. (31) for the PF in a different form by using notations and concepts of the quantum electrodynamics. In terms of these concepts, any state of the system of an electron plus photons is characterized by a state vector $|\Phi(t)\rangle$ which, in the general case, has the form of the above-mentioned superposition

$$|\Phi(t)\rangle=\int d\mathbf{p}\sum_{\{N_{\mathbf{k}}\}} C_{\mathbf{p},\{N_{\mathbf{k}}\}}|\mathbf{p}\rangle|\{N_{\mathbf{k}}\}\rangle \times\exp\left[-it\left(\frac{E_{\mathbf{p}}}{\hbar}+\sum_{\mathbf{k}} N_{\mathbf{k}}\omega_{\mathbf{k}}\right)\right], \tag{66}$$

where the Dirac notation $|\mathbf{p}\rangle$ indicates an electron plane-wave state such that $\langle\mathbf{r}|\mathbf{p}\rangle=\Psi_{\mathbf{p}}(\mathbf{r})$ and $\Psi_{\mathbf{p}}(\mathbf{r})$ is given by Eq. (17). The field-state $|\{N_{\mathbf{k}}\}\rangle$ is the Fock state with definite numbers of photons $N_{\mathbf{k}}$ in all the modes $\{\mathbf{k}\}$, $|\{N_{\mathbf{k}}\}\rangle=\Pi_{\mathbf{k}}|N_{\mathbf{k}}\rangle$, where $|N_{\mathbf{k}}\rangle$ is the $\mathbf{k}$-mode state with a number of photons $N_{\mathbf{k}}$. The sum over $\{N_{\mathbf{k}}\}$ in Eq. (66) means the sum over any possible realizations of numbers of photons in the modes with the weight function determined by the coefficients $C_{\mathbf{p},\{N_{\mathbf{k}}\}}$. The squared averaged vector potential $\mathbf{A}^2$ of Eq. (2), in terms of the photon creation and annihilation operators $\mathbf{a}^{\dagger}_{\mathbf{k}'}$ and $a_{\mathbf{k}}$, is proportional to the sum of their products

$$\overline{\mathbf{A}^2(\mathbf{r},t)}\propto\sum_{\mathbf{k},\mathbf{k}'} a^{\dagger}_{\mathbf{k}'}a_{\mathbf{k}}\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}]. \tag{67}$$

As compared to the exact quantum-electrodynamical operator expression for the squared vector potential, the terms with products of operators $a^{\dagger}_{\mathbf{k}'}a^{\dagger}_{\mathbf{k}}$ and $a_{\mathbf{k}'}a_{\mathbf{k}}$ are dropped or, in other words, only the terms conserving the total number of photons are retained. This last formulation is a quantum-electrodynamical analog of the classical idea about averaging of the squared vector potential over fast time oscillations. In terms of the vector state $|\Phi(t)\rangle$ and the creation and annihilation operators $a^{\dagger}_{\mathbf{k}'}$ and $a_{\mathbf{k}}$, Eq. (31) for the ponderomotive force can be rewritten as

$$\begin{aligned}
\mathbf{F}_{\text{pond}}(t)&\propto\int d\mathbf{r}\left\langle\Phi(t)\left|\sum_{\mathbf{k},\mathbf{k}'}(\mathbf{k}'-\mathbf{k})a^{\dagger}_{\mathbf{k}'}a_{\mathbf{k}}\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}]\right|\Phi(t)\right\rangle\\
&\propto\int d\mathbf{p}\int d\mathbf{p}'\sum_{\{N'_{\mathbf{k}}\}}C^{*}_{\mathbf{p}',\{N'_{\mathbf{k}}\}}\sum_{\{N_{\mathbf{k}}\}}C_{\mathbf{p},\{N_{\mathbf{k}}\}}\delta(\mathbf{p}'-\mathbf{p}-\hbar(\mathbf{k}-\mathbf{k}'))\sum_{\mathbf{k},\mathbf{k}'}(\mathbf{k}'-\mathbf{k})\langle\{N'_{\mathbf{k}}\}|a^{\dagger}_{\mathbf{k}'}a_{\mathbf{k}}|\{N_{\mathbf{k}}\}\rangle\\
&\quad\times\exp\left[it\left(\frac{E_{\mathbf{p}'}-E_{\mathbf{p}}}{\hbar}+\sum_{\mathbf{k}}(N'_{\mathbf{k}}-N_{\mathbf{k}})\omega_{\mathbf{k}}\right)\right]\\
&=\int d\mathbf{p}\int d\mathbf{p}'\sum_{\mathbf{k},\mathbf{k}'}(\mathbf{k}'-\mathbf{k})C^{*}_{\mathbf{p}',N'_{\mathbf{k}}+1,N_{\mathbf{k}}-1}C_{\mathbf{p},N'_{\mathbf{k}},N_{\mathbf{k}}}\sqrt{(N_{\mathbf{k}'}+1)N_{\mathbf{k}}}\,\delta(\mathbf{p}'-\mathbf{p}-\hbar(\mathbf{k}-\mathbf{k}'))\\
&\quad\times\exp\left[it\left(\frac{E_{\mathbf{p}'}-E_{\mathbf{p}}}{\hbar}+\sum_{\mathbf{k}}(N'_{\mathbf{k}}-N_{\mathbf{k}})\omega_{\mathbf{k}}\right)\right],
\end{aligned} \tag{68}$$

where $C_{\mathbf{p},N'_{\mathbf{k}},N_{\mathbf{k}}} \equiv C_{\mathbf{p}\{N_{\mathbf{k}}\}}$, though with only two modes $\mathbf{k}$ and $\mathbf{k}'$ and the corresponding two photon numbers $N_{\mathbf{k}}$ and $N_{\mathbf{k}'}$ indicated explicitly in the notation $C_{\mathbf{p},N_{\mathbf{k}'},N_{\mathbf{k}}}$.

The expressions of Eq. (68) correspond to the above-mentioned interpretation of the ponderomotive force as an averaged diagram of Fig. 1. The integral over $d\mathbf{r}$ and the δ function in Eq. (68) describe the usual momentum conservation rule $[\mathbf{p}'=\mathbf{p}+\hbar(\mathbf{k}-\mathbf{k}')]$. On the other hand, Eq. (68) shows that the elementary processes that give rise to the time-dependent PF do not obey the energy conservation rule: $E_{\mathbf{p}'}$ can be either equal to or not equal to $E_{\mathbf{p}}+\hbar(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'})$. In contrast to SCS, the PF $\mathbf{F}_{\text{pond}}(t)$ ''feels'' an instantaneous field and electron states; it has no memory about the prehistory of the electron and field evolution. However, in the integral characteristics of the ponderomotive scattering, the energy conservation rule can appear. For example, if the scattering problem is formulated as the problem of an incoming and an outgoing electron, the total change of the electron momentum is given by $\Delta\mathbf{p}=\int dt\ \mathbf{F}_{\text{pond}}(t)$. The integral over t gives just the δ function corresponding to the energy conservation rule.

So, finally, one can say that elementary quantum-electrodynamical processes determining PFs are described by the same diagram of Fig. 1 as in the case of SCS. However, in a general case, it is not true that it is only the squared absolute value of this diagram that gives contributions to the probabilities of transitions and rates of change of the electron momentum, etc. This is true only for purely incoherent electron and field states, i.e., for a plane-wave electron state and a field in a Fock state characterized by definite numbers of photons. If, however, both electron and field states are coherent, at least partially, the averaged diagram of Fig. 1 itself can give a nonzero contribution to the force and this is just the ponderomotive force.

As for other general results derived and discussed above, methodically it seems to be very important that, as shown above, both ponderomotive forces and forces arising due to SCS are determined by the ponderomotive potential of an electron potential in an inhomogeneous field. Hence the ponderomotive potential can be considered as a more general characteristic of the electron-field interaction than any specific forces arising under these conditions.

Among the results of specific calculations carried out and described in Sec. IV, it is worth mentioning the estimates of forces $\mathbf{F}_{\text{SCS}}$ as compared to $\mathbf{F}_{\text{pond}}$ and $\mathbf{f}_{\text{SCS}}^{\text{QED}}$. It was shown that, in the quantum-electrodynamical limit, incoherence of the field in pure Fock states can eliminate transverse forces arising due to SCS, whereas the corresponding forces in a classical field remain nonzero. It was assumed that the radiation of a multimode laser can be considered as being intermediate between the cases of a pure Fock state and the classical radiation of a single-mode laser. For this reason, the transverse SCS force experienced by an electron interacting with the field of a multimode laser is expected to be intermediate between $F_{\text{SCS},x}^{\text{cl}}$ of Eq. (48) and $f_{\text{SCS},x}^{\text{QED}}=0$. The difference between the maximal achievable transverse SCS forces in the cases of a single-mode and a multimode lasers can be considered as a measure of incoherence of the radiation of a multimode laser, which partially eliminates the effect of SCS.

It was shown also that in all the cases considered the forces $\mathbf{F}_{\text{SCS}}$ were directed against the ponderomotive forces $\mathbf{F}_{\text{pond}}$. The ratio of these forces $\mathbf{F}_{\text{SCS}}/\mathbf{F}_{\text{pond}}$ was shown to become of the order of one in the field $\varepsilon\sim\varepsilon_c$, where ε_c is given by a solution of Eq. (47). This condition and the field ε_c itself are rather unusual, because they depend explicitly on the geometry of focusing via the parameter L/d. It is not clear whether or not it is possible to extrapolate the results derived upon the case of stronger fields $\varepsilon_0>\varepsilon_c$. Perhaps in the case $\varepsilon_0>\varepsilon_c$ (or $v_\varepsilon>v_0/k_0d$) higher-order perturbation-theory contributions to the forces experienced by an electron in an inhomogeneous field will become so important that any separation for ponderomotive forces and forces arising due to SCS will lose any sense. However, even so, the estimate of Eq. (47) is very important as an estimate of the upper applicability limit of the perturbation theory.

It is worth emphasizing that, although the general equations of Sec. III are free from any limitations on such parameters as the light pulse duration, size of the focus, and size of the electron wave packet, the above-described results of the calculations of Sec. IV were obtained for a very special case, characterized by a series of limitations and simplifying assumptions. In particular, the calculations were carried out in the case of a stationary field and its pulse duration τ was assumed to be much longer than the time of flight of an electron through the focus [Eq. (40)]. This is a restriction for both τ and the electron velocity $v_0=p_0/m$. As for the electron wave function, its size Δr_0 was assumed to be either much smaller or much larger than the focal sizes d and L. In latter of these two cases the electron state was modeled by a plane wave. In the case of a small-size localized electron wave function, its size Δr_0, though small as compared to L, was assumed to be large enough to exclude any spreading effects to arise $[\Delta r(t)\approx\Delta r_0]$ during the time of flight of an electron through the focus [Eq. (42)]. An extension of the above-described calculations upon other cases [when the restrictions of Eqs. (40) and (42) are not fulfilled] is rather important. In particular, it will be very important to consider the case of supershort laser pulses (of femtosecond duration). The results of such calculations will be described elsewhere. Here it should be noted that an example of such an analysis was reported recently in Refs. [16, 17], where reflection of electrons from the evanescent surface wave formed by femtosecond laser pulses was considered.

Among other possible and reasonable extensions of the present consideration, it is worth mentioning the generalization to the relativistic case. In particular, by applying appropriately the procedure of separation of slow and fast motions to the Klein-Gordon equation, we will most likely be able to find relativistic corrections to the ponderomotive potential of Eq. (14). We also plan to consider strong-field effects and multiphoton SCS that can occur when $\varepsilon_0>\varepsilon_c$, where ε_c is a solution of Eq. (47).

Finally, an interesting related problem is the investigation of spontaneous emission of an electron experiencing the action of PFs and forces arising due to SCS and multiphoton SCS. Such processes can play the role of a very convenient tool for investigation of the above-described effects includ-

ing the relationship between the classical and quantum features of both an electron and a field.

As a whole, these and other closely related problems form a research area that can be called electron optics in a strong laser light. In our opinion, this is a rather interesting and promising direction of investigation and we hope to return to the above-outlined problems elsewhere.

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[1] T. W. B. Kibble, Phys. Rev. **150**, 1060 (1966).
[2] J. H. Eberly, Prog. Opt. **7**, 359 (1969).
[3] R. R. Freeman, P. H. Bucksbaum, H. Milchberg, S. Darrak, D. Schumacher, and M. F. Censic, Phys. Rev. Lett. **59**, 1092 (1987).
[4] M. V. Fedorov, in *Interaction of Intense Laser Light with Free Electrons*, edited by V. S. Letokhov, C. V. Shank, Y. R. Shen, and H. Walter (Harwood Academic, London, 1991), Vol. 13.
[5] N. B. Delone and M. V. Fedorov, Prog. Quantum Electron. **13**, 267 (1989).
[6] P. Peyraud, J. Phys. (Paris) **29**, 88 (1968); **29**, 306 (1968); **29**, 876 (1968).
[7] F. V. Bunkin, A. E. Kazakov, and M. V. Fedorov, Sov. Phys. Usp. **15**, 416 (1973).
[8] P. L. Kapitza and P. A. M. Dirac, Proc. Cambr. Philos. Soc. **29**, 297 (1933).
[9] M. V. Fedorov, Zh. Eksp. Teor. Fiz. **52**, 1434 (1967). [Sov. Phys. JETP **25**, 952 (1967)].
[10] P. H. Bucksbaum, D. W. Schumacher, and M. Bashkansky, Phys. Rev. Lett. **61**, 1182 (1988).
[11] A. V. Gaponov and M. A. Miller, Zh. Eksp. Teor. Fiz. **34**, 242 (1958) [Sov. Phys. JETP **7**, 168 (1958)]; **34**, 751 (1958) [**7**, 515 (1958)].
[12] L. D. Landau and E. M. Lifshitz, *Mechanics*, 3rd ed. (Pergamon, Oxford, 1976).
[13] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, Oxford, 1977).
[14] M. V. Fedorov, K. B. Oganesian, and A. M. Prokhorov, Appl. Phys. Lett. **53**, 353 (1988); K. B. Oganesian, A. M. Prokhorov, and M. V. Fedorov, Zh. Eksp. Teor. Fiz. **94**, 80 (1988) [Sov. Phys. JETP **67**, 1342 (1988)].
[15] R. J. Glauber, Phys. Rev. **1**, B2766 (1963).
[16] V. S. Letokhov, Pis'ma Zh. Eksp. Teor. Fiz. **61**, 786 (1995) [JETP Lett. **61**, 805 (1995)].
[17] V. I. Balykin, M. V. Subbotin, and V. S. Letokhov (unpublished).